Opportunity #8: Calculus

Feb. 15, 2017

ANSWERS

Routine Questions

1. If $f(x)=x^2-3x+2$, use the definition of derivative to find $f'(x)$. Verify your result using the power rule.

$$\begin{aligned} f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0}\frac{[(x+h)^2-3(x+h)+2]-[x^2-3x+2]}{h} \\ &= \lim_{h\to 0}\frac{x^2+2xh+h^2-3x-3h+2-x^2+3x-2}{h} \\ &= \lim_{h\to 0}\frac{2xh+h^2-3h}{h} \\ &= \lim_{h\to 0}(2x+h-3) \\ &= 2x-3 \end{aligned}$$

2. Differentiate using the power rule and other shortcut rules. (You do not need to use the definition.)

   a. $\frac{d}{dx}(3x^3+\cos x)$
   $=9x^2-\sin x$

   b. $\frac{d}{dx}\left(\frac{x^2}{2}-\frac{2}{x^2}\right)$
   $=\frac{d}{dx}\left(\frac{1}{2}x^2-2x^{-2}\right)$
   $=x+4x^{-3}$
   $=x+\frac{4}{x^3}$

3. If $f(x)=\sin x$, what is $f^{(5)}(x)$?

$f^{(5)}=\cos x$

4. What can we say about $f'(x)$ when the graph of $f(x)$...
    a. is increasing?
    $f'(x)>0$

    b. is decreasing?
    $f'(x)<0$

    c. reaches the top (local max) of a smooth, continuous curve?
    $f'(x)=0$ ***as the value of*** $f'(x)$ ***changes from positive to negative***

    d. reaches the bottom (local min) of a smooth, continuous curve?
    $f'(x)=0$ ***as the value of*** $f'(x)$ ***changes from negative to positive***

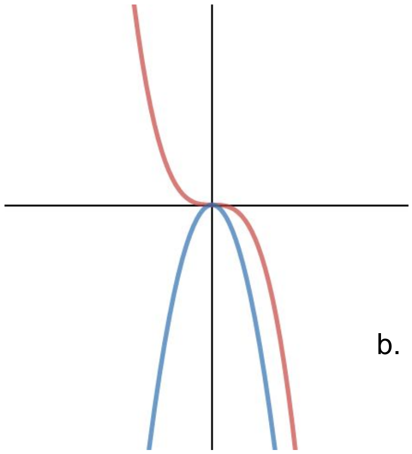
5. The IROC of a function at a certain point, $x=a$, is $\lim_{h\to 0}\frac{[(2+h)^2-(2+h)+3]-5}{h}$.

    a. What is $f(x)$?
    $f(x)=x^2-x+3$

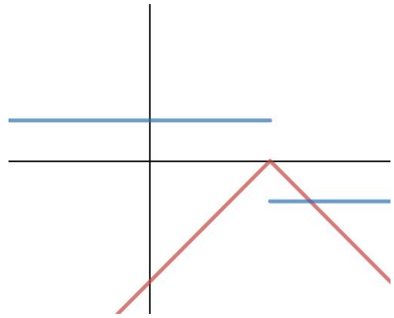
    b. What is the a-value?
    $a=2$

    c. What is $f'(a)$?
    $f'(x)=2x-1$
    $f'(2)=2(2)-1=3$

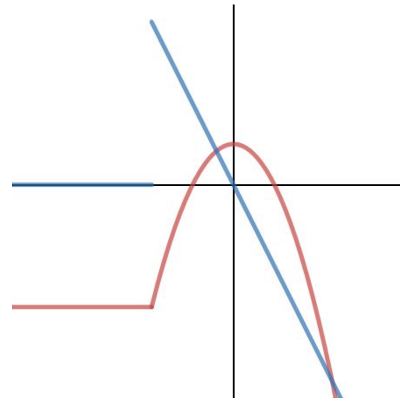
6. The graph of $f(x)$ is shown (next page). For each $f(x)$, sketch $f'(x)$.
    The graph of $f'(x)$ ***is shown in blue.***



a.



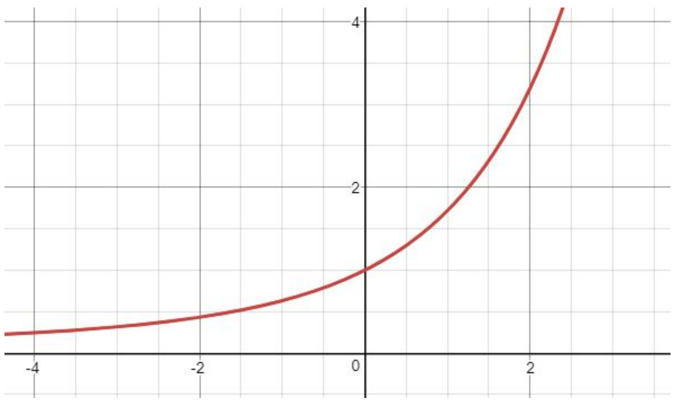
b.



c.

Thinking Questions

7. Consider the graph of $y = \frac{e^x - 1}{x}$, shown below.



   a. What is $\lim_{h \to 0} \frac{e^h - 1}{h}$?

   Based on the graph, and with a change in the letter of the variable, we can see that $\lim_{h \to 0} \frac{e^h - 1}{h} = 1$***.***

   b. Use the properties of exponents to rewrite the expression e^{x+h}.

   $e^{x+h} = e^x e^h$

c. Now use the definition of derivative and the results from (a) and (b) to show that $\frac{d}{dx}e^x = e^x$.

$$
\begin{aligned}
\frac{d}{dx}e^x &= \lim_{h \to 0} \frac{e^{x+h} - e^x}{h} \\
&= \lim_{h \to 0} \frac{e^x e^h - e^x}{h} \\
&= \lim_{h \to 0} \frac{e^x (e^h - 1)}{h} \\
&= e^x \lim_{h \to 0} \frac{e^h - 1}{h} \\
&= e^x (1) \\
&= e^x
\end{aligned}
$$